Exact multipoint and multitime correlation functions of a one-dimensional model of adsorption and evaporation of dimers

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In this work, we provide a method that allows us to compute exactly the multipoint and multitime correlation functions of a one-dimensional stochastic model of dimer adsorption evaporation with random (uncorrelated) initial states. In particular, explicit expressions of the two-point noninstantaneous/instantaneous correlation functions are obtained. The long-time behavior of these expressions is discussed in detail and in various physical regimes.

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One-dimensional reaction-diffusion (RD) processes of *interacting particles* have been extensively studied in the last decade because of their relevance as examples of strongly correlated nonequilibrium systems and their connection with experimental situations [1–4].

Among the RD systems, the "diffusion-limited with pair annihilation and creation" (DPAC) model [5–15] plays a particular role. In fact it is one of the rare nonequilibrium models for which it has been possible, in some special cases, to compute some *dynamical correlation functions*. In addition this model carries valuable information for various experimental situations where particles diffuse and dimer can be adsorbed/evaporated [1,2]. Despite the interest in the DPAC model, not all the desirable information on the correlation functions was available so far. In particular considerably less results are available for the (complete) DPAC model than for the "diffusion-limited pair annihilation" (DPA) model (where there is no pair creation).

Recently, there has been a regain of interest for the study of the DPAC model because its possible application in various fields such as the experimental study of the photogrowth properties of long-lived midgap absorption band in a MX chain [2] and in interdisciplinary studies [3]. In particular, it has been shown that the autocorrelation functions of the DPAC model provide valuable information on the relaxation of biological dimer adsorption [3]. In this work we consider the (free fermion) DPAC and DPA model and obtain results that remained inaccessible so far. In particular, we explicitly compute the exact and complete expression of the noninstantaneous two-point correlation functions for random initial conditions and then analyze the long-time behavior of the latter.

We consider a periodic lattice of L sites (without restriction, L is assumed to be *even*) on which an even number of (classical) particles interact. Each site is either empty or occupied by a particle at most (because of the *hard-core interaction*). When a particle and a vacancies are adjacent to each other, the particle can *jump* to the right with a rate h' or to the left with a rate h. When two particles are adjacent, they can *annihilate in pairs* with a rate ϵ. In addition, when two vacancies are adjacent, a *pair of particles can be created* with rate ϵ'. We now adopt the so-called *stochastic Hamiltonian* formalism (see, e.g., [4] and references therein). To do this, at each of the L lattice sites, we associate to a particle (vacancy) a spin-$\frac{1}{2}$ down (up). In so doing the master equation governing the dynamics of the model can formally be rewritten as an imaginary-time Schrödinger equation for a quantum spin chain: $(\partial/\partial t)|P(t)\rangle = -H|P(t)\rangle$, where $|P(t)\rangle = \Sigma_{\{n\}} P(\{n\},t)|\{n\}\rangle$ describes the state of the system at time t (the sum runs over all the 2^L configurations $\{n\}$). Performing a standard Jordan-Wigner transformation and then a Fourier transformation, the *stochastic Hamiltonian* can be recast in a *fermionic representation* [4–12]. We also define the "left vacuum" $\langle\tilde{\chi}| \equiv \Sigma_{\{n\}}\langle\{n\}|$. The probability conservation yields $\langle\tilde{\chi}|H=0$.

Exact solution of the DPAC model is possible in the *free-fermion case* [4–12], and therefore, with $\gamma \equiv \epsilon + \epsilon' - (h + h')$, one has to impose $\gamma = 0$.

Hereafter, we always consider that the constraint $\gamma = 0$ is fulfilled. In this case, the stochastic Hamiltonian reads

$$H = \sum_{q>0} [\omega(q) a_q^\dagger a_q + \omega^*(q) a_{-q}^\dagger a_{-q} + 2\sin q(\epsilon a_q a_{-q} + \epsilon' a_{-q}^\dagger a_q^\dagger)] + \epsilon' L, \tag{1}$$

where $a_q^\dagger$ and a_q are usual fermion operators. In addition, $\omega(q) \equiv \tilde{c} - b\cos q + iv\sin q$, with $b \equiv \epsilon + \epsilon'$, $\tilde{c} \equiv \epsilon - \epsilon'$, $v \equiv h' - h$, and $q = \pm\pi(2l-1)/L$, $l = 1,2,\dots,L/2$.

We consider translationally invariant and uncorrelated random initial states $|\rho_0\rangle$ with an even number N of particles, of density $\rho_0 = N/L$, i.e.,

$$|\rho_0\rangle = \begin{pmatrix} 1-\rho_0 \\ \rho_0 \end{pmatrix}^{\otimes L}.$$

From the fermion reformulation, an important property of the left vacuum follows: $\langle\tilde{\chi}|a_q^\dagger = \cot(q/2)\langle\tilde{\chi}|a_{-q}$ [4,8,9].

The *free-fermion* character of the stochastic Hamiltonian (1) allows the computation the following zero-time correlators [9], with $\mu \equiv \rho_0/(1-\rho_0)$:

$$\langle\tilde{\chi}|a_{q'}a_q|\rho_0\rangle \equiv \langle a_{q'}a_q\rangle(0) = \frac{\mu^2\cot(q/2)}{1+\mu^2\cot^2(q/2)}\delta_{q,-q'},$$

$$\langle a_{q_1}a_{q_2}\cdots a_{q_{2n-1}}a_{q_{2n}}\rangle(0) = \frac{1}{n!}\sum_\pi \mathcal{S}(\pi)\langle a_{q_{\pi(1)}}a_{q_{\pi(2)}}\rangle \times(0)\cdots\langle a_{q_{\pi(2n-1)}}a_{q_{\pi(2n)}}\rangle(0), \tag{2}$$

where the sum is over all the permutations π of the indices $\{q_1,q_2,\ldots,q_{2n}\}$, with the constraints $\pi(1)<\pi(2),\ldots,\pi(2n-1)<\pi(2n)$. Each permutation π has a signature $\mathcal{S}(\pi)$.

It is advantageous at this point to introduce the "pseudo-fermion" operators [6] $\xi_q=\alpha^{-1}\cos\theta_q a_q+\alpha\sin\theta_q a^{\dagger}_{-q}$ and $\xi^{+}_q=\alpha\cos\theta_q a^{\dagger}_q+\alpha^{-1}\sin\theta_q a_{-q}$, with $\theta_{-q}=-\theta_q$. Although they are *not adjoint* each other [6], these operators fulfill the canonical anticommutation relation $\{\xi_q,\xi^{+}_{q'}\}=\delta_{q,q'}$. The probability conservation yields $\langle\tilde{\chi}|\xi^{+}_q=0$ [6].

It has been shown [6] that if one chooses the following parameters $\tan(2\theta_q)=2\sqrt{\epsilon\epsilon'}\sin q/(b\cos q-\tilde{c})$; $\alpha^2=\sqrt{\epsilon'/\epsilon}$, the stochastic free-fermion Hamiltonian (1) is *diagonal* in the pseudo-fermion representation and reads $H=\sum_q\lambda_q\xi^{+}_q\xi_q$, with $\lambda_q=b-\tilde{c}\cos q+iv\sin q$. Because of this diagonal representation, the pseudofermion operators evolve according to $\xi_q(t)=e^{-\lambda_q t}\xi_q(0)$ and $\xi^{+}_q(t)=e^{\lambda_q t}\xi^{+}_q(0)$.

Using the fact that the pseudo-fermion operators ξ_q are *linear combination* of fermion operators a_q and $a^{\dagger}_q$, and using the expression (2) as well as the property of the left vacuum, it follows that

$$\begin{aligned}\langle\xi_q\xi_{q'}\rangle(0)=&\frac{\cos\theta_q\cos\theta_{q'}}{\alpha^2}\langle a_q a_{q'}\rangle(0)\\&+\alpha^2\sin\theta_q\sin\theta_{q'}\langle a^{\dagger}_{-q}a^{\dagger}_{-q'}\rangle(0)\\&+\sin\theta_{q'}\cos\theta_q\langle a_q a^{\dagger}_{-q'}\rangle(0)\\&+\sin\theta_q\cos\theta_{q'}\langle a^{\dagger}_{-q}a_{q'}\rangle(0).\end{aligned}\tag{3}$$

For the sequel it is useful to compute $\langle\xi_q\xi_{q'}\rangle(0)\equiv\langle\tilde{\chi}|\xi_q\xi_{-q}|\rho_0\rangle\delta_{q',-q}$. Therefore, we introduce the following function:

$$\begin{aligned}f(q)\equiv&\langle\xi_q\xi_{-q}\rangle(0)\\=&\nu_q-\frac{\mu^2\epsilon\nu_q\cos^2(q/2)}{\left(1+\dfrac{\mu^2\epsilon}{\epsilon'}\nu_q^2\right)\mathrm{Re}(\lambda_q)}[1+2\nu_q^2+2/\nu_q^2],\\&\text{where}\quad\nu_q\equiv\sqrt{\epsilon'/\epsilon}\cot(q/2).\end{aligned}\tag{4}$$

One can also check that

$$\begin{aligned}\langle\xi_{q_1}\cdots\xi_{q_{2n}}\rangle(0)=&\frac{1}{n!}\sum_{\pi}\mathcal{S}(\pi)\langle\xi_{q_{\pi(1)}}\xi_{q_{\pi(2)}}\rangle\\&\times(0)\cdots\langle\xi_{q_{\pi(2n-1)}}\xi_{q_{\pi(2n)}}\rangle(0),\end{aligned}\tag{5}$$

where we adopted the same notations as in Eq. (2).

Let us now sketch a four-step procedure that allows us to compute explicitly the *multipoint* and *multitime* correlation functions $\langle n_{j_1}(t_1)\cdots n_{j_{m-1}}(t_{m-1})n_{j_m}(t_m)\rangle\equiv\langle\tilde{\chi}|n_{j_1}e^{-H(t_1-t_2)}\cdots n_{j_{m-1}}\exp[-H(t_{m-1}-t_m)]n_{j_m}e^{-Ht_m}|\rho_0\rangle$.

(i) One first has to write the expression of the correlation functions in the Fourier space.

$$\begin{aligned}\langle n_{j_1}(t_1)\cdots n_{j_m}(t_m)\rangle=&\sum_{q_1,q'_1,\ldots,q_m,q'_m}\exp[i(q_1-q'_1)j_1\\&+\cdots+i(q_m-q'_m)j_m]\\&\times\langle a^{\dagger}_{q_1}(t_1)a_{q'_1}(t_1)\cdots a^{\dagger}_{q_m}(t_m)a_{q'_m}(t_m)\rangle.\end{aligned}\tag{6}$$

(ii) One has then to rewrite the expression (6) in the *pseudofermion* language. This was achieved with the help of

$$\begin{aligned}a^{\dagger}_q a_{q'}=&\cos\theta_q\cos\theta_{q'}\xi^{+}_q\xi_{q'}-\cos\theta_q\sin\theta_{q'}\xi^{+}_q\xi^{+}_{-q'}\\&-\sin\theta_q\cos\theta_{q'}\xi_{-q}\xi_{-q'}+\sin\theta_q\sin\theta_{q'}\xi_{-q}\xi^{+}_{-q'}.\end{aligned}\tag{7}$$

(iii) Using the fact that H is *diagonal* in the (pseudo-fermion) representation [with Eq. (7)], one extracts the time dependence of terms appearing in the pseudofermion rewritten expression (6). As an example, we have

$$\begin{aligned}&\langle\xi_{q_1}(t_1)\xi_{q'_1}(t_1)\cdots\xi_{q_m}(t_m)\xi_{q'_m}(t_m)\rangle\\&=\exp[-(\lambda_{q_1}+\lambda_{q'_1})t_1-\cdots-(\lambda_{q_m}+\lambda_{q'_m})t_m]\\&\quad\times\langle\xi_{q_1}\xi_{q'_1}\cdots\xi_{q_m}\xi_{q'_m}\rangle(0).\end{aligned}\tag{8}$$

(iv) Finally, the zero-time correlation functions of pseudo-fermion operators appearing on the right-hand side (rhs) of (6), after the steps (i)–(iii), are computed with help of the Wick factorization (5) and using $\langle\tilde{\chi}|\xi^{+}_q=0$.

This general four-step procedure provides a systematic method to obtain explicitly, starting from homogeneous random initial conditions, the multipoint and multitime correlation functions of the free-fermion model under consideration here. It is, however, important to notice that the computation of each of these quantities leads to rather complicated technical difficulties. Let us mention that using the domain-wall duality and with help of the generating function studied in [10], we can compute the stationary multipoint correlation functions of the DPAC model from the spin-spin correlation functions of the one-dimensional Ising model with a generalized (biased) Glauber's dynamics [6,10,16]. In fact we can show $(j_m>\cdots>j_1)$ that $\langle n_{j_1}\cdots n_{j_m}\rangle(\infty)=[\rho(\infty)]^m$ [16], where $\rho(\infty)=\sqrt{\epsilon'}/(\sqrt{\epsilon}+\sqrt{\epsilon'})$ is the stationary density of particles. In addition, for a homogeneous system with initial density $\rho_0=1/2$ of particles, because of the quadratic form of the generating function, the expressions of spin-spin correlation functions of the dual of the DPAC model are Pfaffians [10,16]. In this case it is, therefore, possible, via the domain-wall duality [4,10], to sort out the technical complications and explicitly compute the instantaneous multipoint correlation functions. As an example for the DPA model ($\epsilon=h+h'$, $\epsilon'=0$), the long-time behavior ($bt\gg1$) of the three-point correlation functions reads $8\langle n_j(t)n_{j+r_1}(t)n_{j+r_2}(t)\rangle\approx(1+8\{r_1^2+r_1r_2[5(r_2-r_1)-1]\})/1280\pi(\tilde{c}t)^3$, with $r_2>r_1$ [16].

Because of the general technical problems inherent to the computation of the correlation functions, the above-mentioned systematic four-step procedure is, in particular, useful to take into account random initial conditions, which affect the long-time dynamics of the nonuniversal relaxation (when all the rates h, h', ϵ and ϵ' are >0, see below).

To illustrate the difficulties that appear in computing the multipoint and multitime correlation functions (from uncorrelated but random initial states), as well as their importance, one can point out the work of Derrida and Zeitak [17], where these authors obtained the universal distribution of domain sizes of one-dimensional Potts model with zero-temperature Glauber dynamics. This was achieved using the properties of coalescing random walkers to compute the probability of having the same value at time t for N spins located at N distinct and ordered sites, which is related to the distribution of domain sizes. The authors also studied the domain-walls dynamics and, thus, considered the following (free-fermion) RD model $A\emptyset \leftrightarrow \emptyset A$, $AA \to A$, and $AA \to \emptyset$, with reaction rates 1, $(q-2)/(q-1)$, and $1/(q-1)$, respectively. For this RD model, with random (but uncorrelated) initial conditions, the authors of [17] computed the density of particles and the instantaneous two-point correlation functions. It has to be noticed that there exists a similarity transformation (see, e.g., [4], and references therein) that maps the DPA model studied here (with $\epsilon'=0$) onto the RD model considered in [17].

Another important problem where the (stationary) multipoint correlation functions of a free-fermion model play a relevant role is the computation, for the q-state Potts model in the zero-temperature Glauber dynamics, of the exact persistence exponent that gives the fraction of spins $\tilde{\rho}_L(q)$, which have never flipped [18,19]. To compute $\tilde{\rho}_L(q)$, the authors mapped the problem onto an exactly solvable (free fermion) RD model: $A\emptyset \leftrightarrow \emptyset A$ (with reaction rate 1) and $AA \to A$ (with reaction rate 2). In addition it is assumed that a "source" ensures that the origin of the (periodic) lattice is always occupied. Starting from an uncorrelated but random initial state (with the initial site always occupied) denoted $|P''(0)\rangle$, the problem of finding $\tilde{\rho}_L(q)$ reduces to the computation of the following multipoint correlator [18]: $\tilde{\rho}_L(q) = \lim_{t\to\infty}\langle 0|(1+a_L)\cdots(1+a_1)e^{-H't}|P''(0)\rangle$, where the a_j's are fermion operators, $|0\rangle$ is the vacuum ($a_j|0\rangle=0$) and H' is the stochastic Hamiltonian associated to RD model considered in [18,19].

To be specific we now focus on the computation of the connected noninstantaneous two-point correlation functions of the DPAC model for random initial conditions $|\rho_0\rangle$.

Following the above-mentioned four-step procedure [(i)–(iv)], adopting the notation $\phi_q \equiv b - \tilde{c}\cos q$, we obtain, in the thermodynamic limit ($L\to\infty$):

$$
\begin{aligned}
\mathcal{C}_r(t,t_0) &\equiv \langle n_{j+r}(t+t_0)n_j(t_0)\rangle - \rho(t+t_0)\rho(t_0)\\
&= \epsilon\epsilon'\left(\int_0^\pi \frac{dq}{\pi}\sin[qr-vt\sin q]\frac{\sin q}{\phi_q}e^{-\phi_q t}\right)^2 + 4\epsilon\epsilon'\left(\int_0^\pi\frac{dq}{\pi}\cos[qr-vt\sin q]\right.\\
&\quad\times\left.\frac{\sin^2(q/2)}{\phi_q}e^{-\phi_q t}\right)\left(\int_0^\pi\frac{dq}{\pi}\cos[qr-vt\sin q]\frac{\cos^2(q/2)}{\phi_q}e^{-\phi_q t}\right) + 2\epsilon'\sqrt{\epsilon\epsilon'}\left(\int_0^\pi\frac{dq}{\pi}\sin[qr-vt\sin q]\frac{\sin q}{\phi_q}e^{-\phi_q t}\right)\\
&\quad\times\left(\int_0^\pi\frac{dq}{\pi}\sin[qr-vt\sin q]\frac{\cos^2(q/2)}{\phi_q}f(q)e^{-\phi_q(t+2t_0)}\right) - \sqrt{\epsilon\epsilon'}\left(\int_0^\pi\frac{dq}{\pi}\cos[qr-vt\sin q]\frac{\tilde{c}-b\cos q}{\phi_q}e^{-\phi_q t}\right)\\
&\quad\times\left(\int_0^\pi\frac{dq}{\pi}\cos[qr-vt\sin q]\frac{\sin q}{\phi_q}f(q)e^{-\phi_q(t+2t_0)}\right) - 2\epsilon\sqrt{\epsilon\epsilon'}\left(\int_0^\pi\frac{dq}{\pi}\sin[qr-vt\sin q]\frac{\sin q}{\phi_q}e^{-\phi_q t}\right)\\
&\quad\times\left(\int_0^\pi\frac{dq}{\pi}\sin[qr-vt\sin q]\frac{\sin^2(q/2)}{\phi_q}f(q)e^{-\phi_q(t+2t_0)}\right) - 4\epsilon\epsilon'\left(\int_0^\pi\frac{dq}{\pi}\sin[qr-vt\sin q]\frac{\cos^2(q/2)}{\phi_q}\right.\\
&\quad\times\left.f(q)e^{-\phi_q(t+2t_0)}\right)\left(\int_0^\pi\frac{dq}{\pi}\sin[qr-vt\sin q]\frac{\sin^2(q/2)}{\phi_q}f(q)e^{-\phi_q(t+2t_0)}\right) - \epsilon\epsilon'\left(\int_0^\pi\frac{dq}{\pi}\cos[qr-vt\sin q]\right.\\
&\quad\times\left.\frac{\sin q}{\phi_q}f(q)e^{-\phi_q(t+2t_0)}\right)^2.
\end{aligned}
\tag{9}
$$

In this expression $\rho(t)=\sqrt{\epsilon'}/(\sqrt{\epsilon}+\sqrt{\epsilon'}) - \sqrt{\epsilon\epsilon'}\int_0^\pi(dq/\pi)[\sin(q)/\phi_q]f(q)e^{-2\phi_q t}$ designates the (translationally invariant) density of particles at time t. The latter has been previously studied (for the initial states $|\rho_0\rangle$) in [10].

Equation (9) is the main result of this work and provides the complete expression of the noninstantaneous two-point correlation functions of the (free-fermion version of the) DPAC model. From the latter, it is clear that one can also obtain the *instantaneous* two-point connected correlation functions $\mathcal{C}_r(t=0,t_0>0)$, and one can check, as expected from the general properties of the DPAC model [8], that the latter do not depend on the bias v. In addition, it is clear that the expression (9) also includes the instantaneous and noninstantaneous connected two-point correlation functions of the DPA model, where $\epsilon'=0$ and $b=\tilde{c}>0$. It is worthwhile to notice that the noninstantaneous correlation functions (9) [with $t>0$ and $v\neq 0$] depend on the *sign* of r: $\mathcal{C}_r(t,t_0;v)=\mathcal{C}_{-r}(t,t_0;-v)$. Conversely, the *instantaneous* correlation functions [with $t=0$ and $t_0>0$ in Eq. (9)] do *not* depend on the *sign* of r. Let us also stress the fact that setting in Eq. (9) $\mu=\infty$ (i.e., $\rho_0=1$), $\mu=0$ (i.e., $\rho_0=0$), or $\mu=1$ (i.e., $\rho_0=1/2$), we recover results obtained in [6–12].

To proceed with a long-time study of the expression (9), we have carried out a systematic asymptotic expansion of the integrals appearing in Eq. (9), in which the small q regime of integration dominates. Hereafter, we analyze two different regimes and distinguish the case with pair creation (i.e., with $\epsilon'>0$ and $\tilde{c}\neq 0$) from the case with only (asymmetric) diffusion and pair annihilation (i.e., with $\epsilon'=0$ and $\epsilon>0$). The cases where $\tilde{c}=0$ (i.e., $\epsilon=\epsilon'$, $h=h'$, and $\epsilon=\epsilon'$, $h+h'=2\epsilon$) have already been studied in [6].

Regime 1(i) We first consider the case where $\epsilon'>0$ and $\epsilon>0$ in the regime where $\epsilon t,\epsilon t_0\gg 1$ and $\epsilon' t,\epsilon' t_0\gg 1$ [with $\tilde{c}>0$]. In this situation the main contribution to the noninstantaneous correlation function arises from the second term on the rhs of Eq. (9). We, thus, obtain ($v\neq 0$, $\tilde{c}>0$)

$$\mathcal{C}_r(t,t_0)\approx\frac{\epsilon(1-u)e^{-u-4\epsilon' t}}{16\pi\epsilon'(\tilde{c}t)^2},\quad u\equiv(r-vt)^2/\tilde{c}t. \tag{10}$$

It is clear from Eq. (10) that in this regime the late behavior of the noninstantaneous correlation functions $\mathcal{C}_r(t,t_0)$ only depends on the time t (and not on t_0). We notice the nontrivial effect of the bias $v\neq 0$ through the parameter u. In the absence of the bias and for $r<\infty$ (i.e., for the autocorrelation functions) we obtain: $\mathcal{C}_r(t,t_0;v=0)\approx(\epsilon/32\pi\epsilon')\{\exp[-4\epsilon' t]/(\tilde{c}t)^2\}$.

In this regime we now focus on the long-time behavior of the *instantaneous* correlation functions $\mathcal{C}_r(t)$ (obtained setting $t=0$ in Eq. (9) and relabeling t_0 as the variable t). The cases $\rho_0=1$ and $\rho_0=0$ having been studied previously [$\mathcal{C}_r(t;\rho_0=0,1)\propto e^{-4\epsilon' t}t^{-\nu'}$, $\nu'=3/2$ for $\rho_0=1$ and $\nu'=1/2$ for $\rho_0=0$ [6,12]], here we focus on the case of random initial states, i.e., with $0<\mu<\infty$ [and $\rho_0\neq\rho(\infty)=\sqrt{\epsilon'}/(\sqrt{\epsilon'}+\sqrt{\epsilon})$, which would correspond to the *trivial case* where $f(q)\equiv 0$]. The main contribution to $\mathcal{C}_r(t)$ comes from the third and fourth term on the rhs of Eq. (9). Introducing the parameters

$$4A_0\equiv-\sqrt{\epsilon/\epsilon'}(1-\zeta)^2\zeta^{r-1};\quad B_0\equiv-(1-\zeta^2)\zeta^{r-1};$$

$$4C_0\equiv\sqrt{\epsilon'/\epsilon}(1+\zeta)^2\zeta^{r-1};\quad \zeta\equiv\frac{\sqrt{\epsilon}-\sqrt{\epsilon'}}{\sqrt{\epsilon}+\sqrt{\epsilon'}}, \tag{11}$$

we obtain ($r<\infty$)

$$\mathcal{C}_r(t)\approx\frac{\pi e^{-4\epsilon' t}\mathcal{F}(\mu,r,\epsilon,\epsilon')}{4(\pi\tilde{c}t)^{3/2}}, \tag{12}$$

where the rather complicated expression of the amplitude $\mathcal{F}(\mu,r,\epsilon,\epsilon')$ reads

$$\begin{aligned}\mathcal{F}(\mu,r,\epsilon,\epsilon')=&\frac{A_0-C_0}{4\mu^2\sqrt{\epsilon\epsilon'^5}}\{2\mu^2(\epsilon\epsilon')^{3/2}-2\epsilon'^2\sqrt{\epsilon\epsilon'}(\mu r)^2\\&+\epsilon\epsilon'(\sqrt{\epsilon'^3/\epsilon}+\mu^2[2r^2\sqrt{\epsilon'^3/\epsilon}-3\sqrt{\epsilon\epsilon'}])\}\\&-\frac{B_0 r}{12\epsilon\epsilon'^2\mu^2}\{6\mu^2(\epsilon\epsilon')^{3/2}\\&-\mu^2(2r^2+1)\epsilon'^2\sqrt{\epsilon\epsilon'}+\epsilon\epsilon'(3\sqrt{\epsilon'^3/\epsilon}\\&+\mu^2[(2r^2+1)\sqrt{\epsilon'^3/\epsilon}-9\sqrt{\epsilon\epsilon'}])\}.\end{aligned} \tag{13}$$

It is remarkable that conversely to the *instantaneous* correlation functions (12), which amplitude (13) depends on the initial state through the parameter $0<\mu<\infty$, the long-time behavior of the noninstantaneous correlation functions (12) do *not* depend on ρ_0. This is due to the fact that the second term of Eq. (9) does *not* depend on $f(q)$.

Regime 1(ii) Another interesting asymptotic regime to investigate is the one first studied by Torney and McConnell [15], where one considers initially very diluted systems, i.e., $\rho_0\approx\mu\ll 1$, but keeps the products $\epsilon\rho_0^2 t$, $\epsilon\rho_0^2 t_0$, $\epsilon'\rho_0^2 t$, $\epsilon'\rho_0^2 t_0$ fixed and finite, with $\epsilon t,\epsilon t_0\gg 1$ and $\epsilon' t,\epsilon' t_0\gg 1$ [and $\tilde{c}>0$].

In this regime, the noninstantaneous two-point correlation functions $\mathcal{C}_r(t,t_0)$ are still dominated by the second term of Eq. (9) and, thus, the asymptotic ($v\neq 0$) decay of $\mathcal{C}_r(t,t_0)$ is still given by Eq. (10).

The situation is, however, different for the *instantaneous* correlation functions [because the third and fourth term of Eq. (9) depend on $f(q)$]. With help of Eq. (11), we obtain ($r<\infty$)

$$\begin{aligned}\mathcal{C}_r(t)\approx\rho_0^3 e^{-4\epsilon' t}\mathcal{G}(\rho_0,r,\epsilon,\epsilon')\Bigg[&\frac{1}{2(\rho_0^2\pi\tilde{c}t)^{1/2}}\\&-e^{4\rho_0^2\tilde{c}t}\mathrm{erfc}(2\rho_0\sqrt{\tilde{c}t})\Bigg],\end{aligned} \tag{14}$$

where erfc(z) denotes the usual complementary error function. The amplitude $\mathcal{G}(\rho_0,r,\epsilon,\epsilon')$ has the following form:

$$\begin{aligned}\mathcal{G}(\rho_0,r,\epsilon,\epsilon')\equiv&(A_0-C_0)[\rho_0^{-2}-4r^2+\epsilon/\epsilon']\\&-B_0\sqrt{\epsilon'/\epsilon}[r/\rho_0^2-10r(2r^2+1)-\sqrt{\epsilon/\epsilon'}].\end{aligned} \tag{15}$$

We now pass to the case of the DPA model, where $\epsilon'=0$ and $b=\tilde{c}=\epsilon>0$. Again, we distinguish two different regimes.

Regime 2(i) We first consider the regime where $bt \gg 1$ and $bt_0 \gg 1$. In this situation, the main contribution to the long-time dynamics arises from the fourth and the last terms of Eq. (9) and we obtain

$$\mathcal{C}_r(t,t_0;\epsilon'=0) \approx \frac{\exp\left[-\frac{(r-vt)^2}{bt}\frac{t_0+t}{2t_0+t}\right]}{2\pi b\sqrt{t(t+2t_0)}} \times \left(1-\sqrt{\frac{t}{t+2t_0}}e^{[(r-vt)^2/bt]t_0/2t_0+t}\right). \tag{16}$$

It has to be noticed that, according to Eq. (16), when $r>0$ and $v>0$ (or, $r<0$ and $v<0$), $\mathcal{C}_r(t,t_0)$ has a local maximum (a "peak") at time $t_p \equiv r/v$. When $bt \gg bt_0 \gg 1$, we recover the result [7,11] $\mathcal{C}_r(t,t_0;\epsilon'=0) \approx t_0 \exp[-(r-vt)^2/bt]/2\pi bt^2$.

In this regime, the main contribution to the *instantaneous* correlation function arises from the last term of Eq. (9), and we obtain the following result:

$$\mathcal{C}_r(t;\epsilon'=0) \approx -1/4\pi bt, \tag{17}$$

where the minus sign manifests the fact that the long-time dynamics of the DPA model is dominated by *anticorrelation*, due to the pair annihilation of the particles. The results (16) and (17) and the fact that the latter do *not* depend on ρ_0 confirm, for random initial case, the universal character of the DPA model in this regime.

Regime 2(ii) We now consider the low-density regime of the DPA model, where $\rho_0 \approx \mu \ll 1$ and $\epsilon t, \epsilon t_0 \gg 1$, with $\epsilon\rho_0^2 t$ and $\epsilon\rho_0^2 t_0$ finite. Also in this regime the main contribution to $\mathcal{C}_r(t,t_0;\epsilon'=0)$ arises from the fourth and the last terms of Eq. (9) and one has the long-time behavior ($v=0$ and $r<\infty$)

$$\mathcal{C}_r(t,t_0;\epsilon'=0) \approx \rho_0 \exp[2\rho_0^2 b(t+2t_0)] \times \operatorname{erfc}[2\rho_0\sqrt{b(t_0+t/2)}]\left\{\frac{1}{\sqrt{2\pi bt}} - \rho_0 \exp[2\rho_0^2 b(t+2t_0)] \times \operatorname{erfc}[2\rho_0\sqrt{b(t_0+t/2)}]\right\}. \tag{18}$$

For the *instantaneous* correlation functions, we obtain the following long-time behavior:

$$\mathcal{C}_r(t;\epsilon'=0) \approx -\rho_0^2 e^{8\rho_0^2 bt}[\operatorname{erfc}(2\rho_0\sqrt{bt})]^2, \tag{19}$$

where the *anticorrelated* character of the DPA model clearly appears.

Despite the fact that the parameter $\tilde{c}=\epsilon-\epsilon'$ can take negative values, so far we have always considered the case where $\tilde{c}>0$. With help of the similarity transformation $\mathcal{B}=\Pi_{j=1}^{L}\sigma_j^x$, where σ_j^x is the usual Pauli's matrix acting on the site j, we show that the case where $\tilde{c}<0$ is directly related to the one where $\tilde{c}>0$. In fact, according to $\mathcal{B}$, the (free fermion) DPAC stochastic Hamiltonian $H=H(b,\tilde{c},v)$ is mapped onto $\mathcal{B}H(b,\tilde{c},v)\mathcal{B}^{-1}=H(b,-\tilde{c},-v)$ and the initial state $|\rho_0\rangle$ is mapped onto $\mathcal{B}|\rho_0\rangle=|1-\rho_0\rangle$. Therefore, we have $\mathcal{C}_r(t,t_0)_{b,-\tilde{c},-v;\rho_0}=\mathcal{C}_r(t,t_0)_{b,\tilde{c},v;1-\rho_0}$.

In summary, in this work we sketch a four-step procedure that allows the explicit computation of the multipoint and multitime correlation functions of the free-fermion DPAC model starting from random (uncorrelated) initial states. We then specifically compute the noninstantaneous/instantaneous two-point correlation functions in the presence as well as in the absence of the pair-creation term. When all the reaction rates are positive, the dynamics turns out to be nonuniversal and the long-time relaxation is exponential (with a subdominant a power-law factor): the amplitude of the instantaneous two-point correlation functions depends on the initial density ρ_0 and is explicitly determined. In the absence of the pair creation, i.e., when $\epsilon'=0$ and $h+h'=\epsilon>0$, the dynamics turns out to be universal (in the regime where ρ_0 is finite and $bt \gg 1$, $bt_0 \gg 1$) and there is a power-law relaxation. The effect of the bias $v=h'-h\neq 0$, only appears in the noninstantaneous correlation functions and can be absorbed (for $\epsilon' \geqslant 0$) in a Galilean transformation, as noticed in [7,11] in considering the DPA model (in these previous works, $\epsilon'=0$).

To close this work, it is natural to wonder what is the effect on the dynamics of the restriction $\gamma=0$. In fact, it is by now well established on the basis of numerous consistent numerical results [3,6,11,14], and from comparison with experiments [1,2], that the results obtained for the free-fermion version of the DPAC model give a *qualitative* picture that is still valid when $\gamma\neq 0$. One can, therefore, expect that the results obtained in this work could have a general validity and, in particular, a direct relevance for recent interdisciplinary studies [3].

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[1] *Nonequilibrium Statistical Mechanics in One Dimension*, edited by V. Privman (Cambridge University Press, Cambridge, England, 1997).

[2] N. Kuroda, M. Nishida, Y. Tabata, Y. Wakabayashi, and K. Sasaki, Phys. Rev. B **61**, 11 217 (2000).

[3] A. Vilfan, E. Frey, and F. Schwabl, Europhys. Lett. **56**, 420 (2001).

[4] G. M. Schütz, in *Phase Transitions and Critical Phenomena*, edited by C. Domb and J. Lebowitz (Academic Press, London, 2000), Vol. 19.

[5] A. A. Lushnikov, Zh. Eksp. Teor. Fiz. **91**, 1376 (1986) [Sov. Phys. JETP **64**, 811 (1986)].
[6] M. D. Grynberg, T. J. Newman, and R. B. Stinchcombe, Phys. Rev. E **50**, 957 (1994); M. D. Grynberg and R. B. Stinchcombe, Phys. Rev. Lett. **74**, 1242 (1995); Phys. Rev. E **52**, 6013 (1995); M. D. Grynberg, *ibid.* **57**, 74 (1998).
[7] G. M. Schütz, Phys. Rev. E **53**, 1475 (1996).
[8] G. M. Schütz, J. Phys. A **28**, 3405 (1995).
[9] J. Santos, G. M. Schütz, and R. B. Stinchcombe, J. Chem. Phys. **105**, 2399 (1996).
[10] M. Mobilia and P.-A. Bares, Phys. Rev. E **63**, 056112 (2001).
[11] M. D. Grynberg and R. B. Stinchcombe, Phys. Rev. Lett. **76**, 851 (1996).
[12] P.-A. Bares and M. Mobilia, Phys. Rev. E **59**, 1996 (1999).
[13] M. J. de Oliveira, Phys. Rev. E **60**, 2563 (1999).
[14] P.-A. Bares and M. Mobilia, Phys. Rev. Lett. **83**, 5214 (1999); **85**, 893 (2000); S. C. Park, J.-M. Park, and D. Kim, *ibid.* **85**, 892 (2000); Phys. Rev. E **63**, 057102 (2001).
[15] D. C. Torney and M. McConnell, J. Phys. Chem. **87**, 1941 (1983).
[16] M. Mobilia (unpublished).
[17] B. Derrida and R. Zeitak, Phys. Rev. E **54**, 2513 (1996).
[18] B. Derrida, V. Hakim, and V. Pasquier, J. Stat. Phys. **85**, 763 (1996).
[19] B. Derrida, V. Hakim, and V. Pasquier, Phys. Rev. Lett. **75**, 751 (1995).